# Gauge-Consistent Effective Currents in Five-Dimensional Projection Electromagnetism

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**Abstract**

Five-dimensional projection models of electromagnetism often produce a four-dimensional Maxwell equation with an additional effective current. The difficulty is not writing such a current; the difficulty is writing one that is compatible with four-dimensional gauge invariance, charge conservation, and exact recovery of ordinary Maxwell theory in an appropriate limit. This paper formulates a gauge-consistent effective-current framework for speculative five-dimensional projection electromagnetism. Starting from a five-dimensional Maxwell-like system on coordinates $X^A = (x^\mu, \chi)$, we derive the projected equation

$$\nabla_\mu F^{\mu\nu} = \mu_0 \left(J^\nu + J^\nu_{\text{new}}\right), \tag{0.1}$$

and impose local admissibility conditions on $J^\nu_{\text{new}}$. On a contractible four-dimensional patch, every smooth identically conserved correction can be written as the divergence of an antisymmetric polarization tensor, up to singular or boundary-supported terms. This reorganizes a broad class of speculative five-dimensional branches into a small number of operator families: scalar weighting, fifth-gradient flow, projection-curvature mixing, mixed-normal flux, and higher-gradient boundary response. The result is not a complete five-dimensional theory. It is a referee-facing effective dictionary: it identifies which projected currents are admissible, which branches are observationally degenerate, and which experimental parity tests can distinguish surviving operator classes.

## 1 Introduction

The source taxonomy in Ref. [1] proposes twelve possible interpretations of a fifth coordinate in a projection-based electromagnetic theory. Ref. [2] then narrows the discussion to electrostatic couplings that might, in principle, be read as electrogravity-like residuals. Both papers share a useful discipline: ordinary Maxwell theory must be recovered exactly when the fifth-sector fields are suppressed or decoupled. This recovery criterion is more important than any single speculative branch, because it separates harmless notation from physical deformation.

The central object is an additional projected current, $J^\nu_{\text{new}}$, appearing in the four-dimensional inhomogeneous Maxwell equation. In informal speculative writing, such a term can be made to represent almost anything: hidden charge, motion in a compact direction, scalar response, hypersurface curvature, or normal flux through an embedded four-manifold. In a publishable theory paper, however, the same term must obey strict consistency conditions. If it violates gauge invariance, creates or destroys charge without an explicit reservoir, or fails to vanish in the Maxwell limit, the model has not extended electrodynamics; it has merely broken it.

The purpose of this paper is therefore modest and structural. We do not claim that the fifth coordinate exists, that the twelve branches are physically realized, or that an electrogravity force has been detected. We ask a sharper question: what local forms can $J^\nu_{\text{new}}$ take if it is to be an admissible four-dimensional effective current induced by a higher-dimensional projection?

The answer is a compact operator basis. A large family of five-dimensional stories collapses into a small number of four-dimensional constitutive archetypes. This gives the program a path toward publication: replace broad metaphysical branch claims with explicit operators, coefficients, recovery limits, and parity signatures.

## 2 Five-dimensional starting point

Let $(\mathcal{M}_5, G_{AB})$ be a five-dimensional manifold with coordinates $X^A = (x^\mu, \chi)$, where $\mu, \nu = 0, 1, 2, 3$ and $A, B = 0, 1, 2, 3, 5$. Let $\mathcal{A}_A$ be a five-dimensional gauge potential and

$$\mathcal{F}_{AB} = \nabla_A \mathcal{A}_B - \nabla_B \mathcal{A}_A \tag{2.1}$$

be the five-dimensional field strength. The five-dimensional source equation is written

$$\nabla_A \mathcal{F}^{AB} = \mu_5 \mathcal{J}^B. \tag{2.2}$$

A four-dimensional observer sees the tangential components

$$F_{\mu\nu} = \mathcal{F}_{\mu\nu}, \qquad V_\mu = \mathcal{F}_{\mu 5}, \tag{2.3}$$

plus possible metric and embedding data. Splitting Eq. (2.2) along the four-dimensional directions gives the schematic projected equation

$$\nabla_\mu F^{\mu\nu} + \nabla_5 \mathcal{F}^{5\nu} + \Gamma \cdot \mathcal{F} = \mu_5 \mathcal{J}^\nu, \tag{2.4}$$

where $\Gamma \cdot \mathcal{F}$ denotes connection and projection terms. After normalization to four-dimensional units, the result can be written as

$$\nabla_\mu F^{\mu\nu} = \mu_0 \left(J^\nu + J^\nu_{\text{new}}\right). \tag{2.5}$$

The term $J^\nu$ is the conventional electromagnetic four-current. The correction $J^\nu_{\text{new}}$ is the object to be classified.

The homogeneous equation must remain

$$\nabla_{[\alpha} F_{\beta\gamma]} = 0, \tag{2.6}$$

whenever $F$ is the pullback or tangential component of an exact higher-dimensional two-form. This requirement is non-negotiable: if $F = \mathrm{d}A$ locally, then Eq. (2.6) follows independent of interpretation.

## 3 Admissibility criteria

We impose four conditions on an admissible projected current.

**Gauge compatibility.** The correction must be expressible in terms of gauge-invariant fields such as $F_{\mu\nu}$, $V_\mu$ when gauge invariant, scalar moduli, embedding curvature, or covariant derivatives thereof. It must not depend on $A_\mu$ in a way that changes under $A_\mu \mapsto A_\mu + \partial_\mu \lambda$, unless the dependence occurs inside a gauge-fixed auxiliary sector with compensating degrees of freedom.

**Charge consistency.** Applying $\nabla_\nu$ to Eq. (2.5) gives

$$\nabla_\nu \nabla_\mu F^{\mu\nu} = 0 \tag{3.1}$$

for an antisymmetric field strength on a torsion-free connection. Therefore

$$\nabla_\nu \left(J^\nu + J^\nu_{\text{new}}\right) = 0. \tag{3.2}$$

If the ordinary current is separately conserved, then

$$\nabla_\nu J^\nu_{\text{new}} = 0. \tag{3.3}$$

If ordinary charge can leak into the fifth sector, this equation is replaced by an explicit exchange law, not by silent nonconservation.

**Maxwell recovery.** There must be a specified limiting subspace $\mathcal{L}_M$ on which

$$J^{\nu}_{\text{new}}\big|_{\mathcal{L}_M} = 0. \tag{3.4}$$

Typical sufficient conditions are $\partial_\chi F_{AB} = 0$, $V_\mu = 0$, constant scalar modulus, vanishing normal flux, and vanishing projection-curvature contractions.

**Dimensional closure.** Every term must carry the units of current density. When the fifth coordinate is not a length, a conversion scale must be declared. Hidden dimensional conversion is a frequent source of false novelty in speculative extensions.

# 4 A local decomposition theorem

A useful theorem organizes admissible corrections. On a contractible patch of four-dimensional spacetime, suppose $J^{\nu}_{\text{new}}$ is a smooth vector density satisfying Eq. (3.3) identically, rather than only after using special equations of motion. Then there exists an antisymmetric tensor $\mathcal{P}^{\mu\nu} = -\mathcal{P}^{\nu\mu}$ such that

$$J^{\nu}_{\text{new}} = \nabla_\mu \mathcal{P}^{\mu\nu}. \tag{4.1}$$

Boundary-supported and singular contributions may be added separately:

$$J^{\nu}_{\text{new}} = \nabla_\mu \mathcal{P}^{\mu\nu} + J^{\nu}_{\text{bdry}}, \qquad \nabla_\nu J^{\nu}_{\text{bdry}} = 0 \tag{4.2}$$

under the appropriate distributional interpretation.

The proof is the covariant version of the Poincare lemma for conserved currents. In differential-form notation, a conserved current is represented by a closed three-form $\jmath_{\text{new}}$. On a contractible patch, $\jmath_{\text{new}} = \mathrm{d}p$ for some two-form $p$. Translating back to tensor notation gives Eq. (4.1). The result does not say that all microscopic theories are identical. It says that, locally and at the level of Maxwell's source equation, smooth conserved corrections enter as effective polarization-magnetization response.

Equation (4.1) is powerful because it turns projected electromagnetism into a constitutive problem. The modified Maxwell equation can be moved into the form

$$\nabla_\mu \left(F^{\mu\nu} - \mu_0 \mathcal{P}^{\mu\nu}\right) = \mu_0 J^{\nu}. \tag{4.3}$$

Thus many proposed fifth-dimensional source terms are not new charges in the ordinary sense. They are effective medium terms induced by hidden geometry or hidden fields.

# 5 Low-order operator basis

At low derivative order, an admissible polarization tensor can be expanded as

$$\begin{aligned}\mathcal{P}^{\mu\nu} &= c_s\, sF^{\mu\nu} + c_\chi \ell_\chi\, \partial_\chi F^{\mu\nu} + c_V \ell_V \left(\nabla^\mu V^\nu - \nabla^\nu V^\mu\right) \\ &\quad + c_K \ell_K \left(K^\mu V^\nu - K^\nu V^\mu\right) + c_{KF} \ell_K^2 K^{[\mu}{}_\alpha F^{\nu]\alpha} + c_h \ell_h^2 \Box F^{\mu\nu} + \cdots .\end{aligned} \tag{5.1}$$

Here $s$ is a scalar modulus or radion-like projection variable, $V_\mu = \mathcal{F}_{\mu 5}$ is a mixed field, $K_{\mu\nu}$ denotes an extrinsic or projection-curvature tensor, and $\ell_i$ are conversion lengths or effective scales. The ellipsis contains higher-derivative terms, nonlinear terms in $F$, and terms requiring additional symmetry assumptions.

The basis separates several branches that are rhetorically different but observationally similar.

| Operator family | Representative term | Primary signature |
|---|---|---|
| Scalar weighting | $sF^{\mu\nu}$ | even in voltage for electrostatic energy sourcing |
| Fifth-gradient flow | $\partial_\chi F^{\mu\nu}$ | dependence on hidden-profile boundary condition |
| Mixed-normal field | $\nabla^{[\mu}V^{\nu]}$ | possible orientation dependence |
| Curvature mixing | $K^{[\mu}{}_{\alpha}F^{\nu]\alpha}$ | enhancement near edges or bent embeddings |
| Higher-gradient response | $\Box F^{\mu\nu}$ | short-range or boundary-layer scaling |

The table also clarifies what cannot be claimed. A measured correction to Gauss's law does not uniquely identify a fifth coordinate. It identifies an operator class. Microscopic interpretation must come later.

# 6 Compression of the twelve branches

The twelve-branch source taxonomy in Ref. [1] can be compressed into fewer four-dimensional archetypes. Compact Kaluza-Klein branches usually generate a gauge field plus a scalar modulus. Charge-fiber branches often reduce to mixed-normal or fifth-gradient terms. Energy-coordinate and entropy-coordinate branches, if they are to be covariant, must appear through scalar fields or preferred-frame structures. Scale-coordinate branches naturally resemble radion weighting. Full-metric branches contain all of the above plus stability constraints. Constraint-hypersurface branches produce boundary and curvature terms.

This compression is not a loss. It is a gain in falsifiability. Instead of testing twelve stories, one tests a smaller number of response forms:

$$J^\nu_{\rm new} = J^\nu_s + J^\nu_\chi + J^\nu_V + J^\nu_K + J^\nu_{\partial^2} + J^\nu_{\rm bdry}. \tag{6.1}$$

A branch that does not produce a distinct operator has no distinct low-energy phenomenology.

# 7 Electrostatic limit and parity diagnostics

In the static limit, write $F^{i0} = E^i/c$ up to convention. The modified Gauss law becomes

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0} + \frac{\rho_{\rm new}}{\epsilon_0}, \qquad \rho_{\rm new} = J^0_{\rm new}/c. \tag{7.1}$$

For scalar weighting,

$$\rho_s \sim \epsilon_0 \nabla\cdot(s\mathbf{E}). \tag{7.2}$$

If $s$ is sourced by $E^2$, then $s$ is even under $V \mapsto -V$ and $\rho_s$ is odd at the local source level but can generate even-in-voltage energy-density observables. For a direct fifth-current term,

$$\rho_\chi \sim \epsilon_0 \ell_\chi \partial_\chi(\nabla\cdot\mathbf{E}) \tag{7.3}$$

or an equivalent hidden-profile term. For curvature mixing,

$$\rho_K \sim \epsilon_0 \ell_K K_i E^i, \tag{7.4}$$

which is naturally edge and geometry sensitive.

These parity and geometry diagnostics are minimal. They do not prove a five-dimensional origin; they determine which operator class remains viable after conventional electromagnetic, thermal, leakage, vibration, and electrohydrodynamic effects are removed.

## 8 Referee-facing limitations

This paper deliberately avoids three claims. First, it does not specify a unique five-dimensional action. Second, it does not claim that any anomalous capacitor force exists. Third, it does not identify the fifth coordinate with a physical observable without a normalization map. These omissions are not defects of the present paper; they are boundaries. A classification paper should not pretend to solve the microscopic theory.

The main unresolved task is global. The local decomposition theorem can fail globally when the four-dimensional patch has nontrivial cohomology or when singular sources are present. In that case, boundary and topological terms require separate treatment. This is precisely why boundary-flux branches deserve their own paper.

## 9 Conclusion

Five-dimensional projection electromagnetism becomes scientifically useful only after its projected current is constrained. Gauge invariance, charge consistency, dimensional closure, and exact Maxwell recovery force $J^{\nu}_{\rm new}$ into a limited set of effective-current forms. The twelve speculative branches are therefore better understood as microscopic interpretations of a smaller operator basis. This translation from branch language to operator language is the first step toward publishable speculative physics: it replaces broad uncharted possibility with explicit equations, degeneracies, and tests.

## References


[1] JMP0X1B Research, "Twelve 5D projection paths for electromagnetism," research.jmp0x1b.com/papers/twelve-5d-projection-electromagnetism.html.

[2] JMP0X1B Research, "Electrostatic projection couplings in 5D electrogravity," research.jmp0x1b.com/papers/electrostatic-projection-couplings-5d-electrogravity.html.

[3] T. Kaluza, "Zum Unitatsproblem der Physik," Sitzungsberichte Preussische Akademie der Wissenschaften, 1921.

[4] O. Klein, "Quantentheorie und funfdimensionale Relativitatstheorie," Zeitschrift fur Physik, 1926.

[5] A. Einstein and P. Bergmann, "On a generalization of Kaluza's theory of electricity," Annals of Mathematics, 1938.

[6] J. M. Overduin and P. S. Wesson, "Kaluza-Klein gravity," Physics Reports 283, 303-378, 1997.

[7] P. S. Wesson, Space-Time-Matter, World Scientific, 1999.

[8] T. Liko, "Effective electromagnetic current in Kaluza-Klein theory," arXiv:0705.2252.

[9] M. Cruz, F. Dahia, and C. Romero, "Induced electromagnetic currents in higher-dimensional theories," arXiv:0805.4479.

[10] A. Wang, "Kaluza-Klein reduction and Gauss-Codazzi-Ricci equations," arXiv:hep-th/0505049.

[11] J. M. Baptista, "Kaluza-Klein theory and test particles from Riemannian submersions," arXiv:2306.01049.

[12] T. Collion and A. Vaugon, “A geometric formulation of Kaluza-Klein dynamics,” arXiv:2406.09503.