Boundary-Flux Electromagnetism on Embedded Four-Manifolds and Null Observables for Capacitor Residuals

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Abstract

One branch of five-dimensional projection electromagnetism treats observed four-dimensional electrodynamics as the pullback of a higher-dimensional field to an embedded hypersurface. This paper develops the linearized boundary-flux version of that idea. Let the observed world be an embedded four-manifold $\Sigma \subset \mathcal{M}_5$ with unit normal n^A and induced metric $h_{AB} = G_{AB} - \sigma n_A n_B$. A five-dimensional field strength decomposes as $\mathcal{F} = \bar{F} + n \wedge \Pi$, where $\bar{F}$ is tangential and Π contains mixed normal components. Pullback of the five-dimensional Bianchi identity yields the homogeneous Maxwell equation on Σ. Projection of the source equation yields an inhomogeneous law with normal-flux and embedding-curvature terms,

$$D_\mu \bar{F}^{\mu\nu} = \mu_0 \bar{J}^\nu + \partial_\perp \Pi^\nu + K_\mu \bar{F}^{\mu\nu} + \mathcal{Q}^\nu[K, \Pi] + \cdots . \tag{0.1}$$

In electrostatics this becomes a modified Gauss law with edge-sensitive effective charge densities. We derive dimensionless null observables - pressure, polarity, symmetry, inversion, and shielding ratios - that distinguish boundary-flux residuals from electrohydrodynamic, leakage, thermal, and scalar-energy effects. The paper does not claim an anomalous capacitor force. It gives the boundary-flux branch a mathematical form and a falsification protocol.

1 Introduction

The constraint-hypersurface branch in Ref. [1] is among the most geometrically distinctive parts of the five-dimensional projection program. Instead of treating the fifth coordinate as a compact physical circle, scalar scale, entropy coordinate, or charge fiber, it treats four-dimensional physics as the result of restricting a higher-dimensional field to a hypersurface. In such a picture, new source terms may appear when normal flux, extrinsic curvature, or boundary conditions fail to vanish.

This branch deserves separate treatment because it is easy to state vaguely and difficult to test. A phrase such as “flux through the fifth direction” is not yet physics. It becomes physics only when one specifies what is pulled back, what is projected, which terms survive, and how their signatures differ from conventional electromagnetic artifacts.

This paper provides a linearized version of that program. We formulate the embedded geometry, decompose the field strength, derive the projected source law, reduce it to electrostatics, and build null observables that an experiment can report even when no anomaly is found.

2 Embedded hypersurface geometry

Let $(\mathcal{M}_5, G_{AB})$ be a five-dimensional manifold and let Σ be a four-dimensional embedded hypersurface with embedding map $\iota : \Sigma \to \mathcal{M}_5$. The tangent basis is

$$e^A_\mu = \frac{\partial X^A}{\partial x^\mu}, \tag{2.1}$$

and the induced metric is

$$h_{\mu\nu} = G_{AB} e^A_\mu e^B_\nu. \tag{2.2}$$

Let n^A be a unit normal, $G_{AB} n^A n^B = \sigma = \pm 1$, and define the extrinsic curvature

$$K_{\mu\nu} = e^A_\mu e^B_\nu \nabla_A n_B. \tag{2.3}$$

The intrinsic covariant derivative on Σ is denoted D_μ.

For intuition, one may write a graph embedding $\chi = f(x^\mu)$ in a coordinate chart. Then derivatives of f determine the local tilt of the hypersurface, while second derivatives determine extrinsic curvature. The formalism below does not require that special chart, but the chart helps explain why edges, boundaries, and sharp electrode geometry can matter.

3 Field decomposition

Let $\mathcal{F}_{AB}$ be a five-dimensional two-form. Its tangential pullback is

$$\bar{F}_{\mu\nu} = e^A_\mu e^B_\nu \mathcal{F}_{AB}. \tag{3.1}$$

The mixed normal component is

$$\Pi_\mu = e^A_\mu n^B \mathcal{F}_{AB}. \tag{3.2}$$

In form notation,

$$\mathcal{F} = \bar{F} + n^\flat \wedge \Pi \tag{3.3}$$

near the hypersurface, where $n^\flat$ is the one-form associated with n^A.

If $\mathcal{F} = \mathrm{d}\mathcal{A}$, the five-dimensional Bianchi identity $\mathrm{d}\mathcal{F} = 0$ holds. Pulling it back to Σ gives

$$\mathrm{d}_\Sigma \bar{F} = 0, \tag{3.4}$$

or in components,

$$D_{[\alpha} \bar{F}_{\beta\gamma]} = 0. \tag{3.5}$$

Thus the homogeneous Maxwell equation is automatically protected by the pullback. Any viable boundary-flux theory must preserve Eq. (3.5).

4 Projected source equation

Start from

$$\nabla_A \mathcal{F}^{AB} = \mu_5 \mathcal{J}^B. \tag{4.1}$$

Projecting tangentially with e^ν_B yields, schematically,

$$D_\mu \bar{F}^{\mu\nu} = \mu_0 \bar{J}^\nu + \mathcal{S}^\nu_\perp + \mathcal{S}^\nu_K + \mathcal{S}^\nu_{K\Pi} + \cdots . \tag{4.2}$$

The leading terms are

$$\mathcal{S}^\nu_\perp \sim \partial_\perp \Pi^\nu, \qquad \mathcal{S}^\nu_K \sim K_\mu \bar{F}^{\mu\nu}, \qquad \mathcal{S}^\nu_{K\Pi} \sim K^\nu{}_\mu \Pi^\mu - K\Pi^\nu, \tag{4.3}$$

where $K = K^\mu{}_\mu$ and $\partial_\perp = n^A \nabla_A$. The precise coefficients depend on normalization, signature, and the chosen foliation. What is invariant at this stage is the structure: tangential divergence receives contributions from normal variation and embedding curvature.

Equation (4.2) must still satisfy a conservation identity. If $\bar{J}^\nu$ is the ordinary conserved current, then the sum of the projected source terms must be divergence-free on Σ, or else it must be accompanied by an explicit exchange current with the bulk:

$$D_\nu \left(\mathcal{S}^\nu_\perp + \mathcal{S}^\nu_K + \mathcal{S}^\nu_{K\Pi} + \cdots \right) = 0 \tag{4.4}$$

for a closed effective system.

5 Electrostatic reduction

Set $\bar{F}_{i0} = E_i/c$ and assume static fields. The projected source equation becomes

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} + \frac{\rho_\perp}{\epsilon_0} + \frac{\rho_K}{\epsilon_0} + \frac{\rho_{K\Pi}}{\epsilon_0} + \cdots . \tag{5.1}$$

A minimal parameterization is

$$\rho_\perp = \epsilon_0 a_\perp \partial_\perp \Pi^0, \tag{5.2}$$

$$\rho_K = \epsilon_0 a_K K_i E^i, \tag{5.3}$$

$$\rho_{K\Pi} = \epsilon_0 a_{K\Pi} \left(K^0{}_\mu \Pi^\mu - K\Pi^0 \right). \tag{5.4}$$

The coefficients a_i carry the conversion factors needed to translate the fifth-direction normalization into ordinary SI units.

The curvature term $K_i E^i$ is especially important. It vanishes in ideal flat symmetric regions and becomes large near geometry changes, edges, boundaries, or effective embedding distortions. Therefore a boundary-flux residual should be edge-localized rather than volume-uniform. This differs from a scalar-energy model, where the source is E^2 throughout the high-field region.

6 Force budget

A measured force in a capacitor-like experiment can be written

$$F_{\text{obs}} = F_{\text{EHD}} + F_{\text{Maxwell}} + F_{\text{thermal}} + F_{\text{leak}} + F_{\text{vib}} + F_{\text{support}} + F_{\text{BF}}, \tag{6.1}$$

where F_{BF} denotes a possible boundary-flux residual. A boundary-flux theory is scientifically meaningful only if it predicts transformations that separate F_{BF} from the other terms.

The basic scaling expectations are:

Contribution	Typical diagnostic	Expected behavior
Electrohydrodynamic	pressure ladder	collapses in high vacuum
Leakage/current	current correlation	tracks I or discharge events
Thermal	delayed response	follows heating and time constants
Maxwell stress	geometry and grounding	reproduced by conventional field simulation
Scalar energy	polarity even, volume energy	scales with E^2 and energy density
Boundary flux	edge and embedding sensitivity	enhanced by edges and null in matched symmetry

7 Null observables

We define five dimensionless ratios. A good null experiment should report them even when the result is compatible with zero.

Pressure ratio.

$$R_p = \frac{F(P_{\text{low}})}{F(P_{\text{atm}})}. \tag{7.1}$$

Electrohydrodynamic effects generally decrease strongly with pressure, while a genuine boundary-flux term tied only to fields and geometry should not follow the same gas-density scaling.

Polarity ratio.

$$R_{\pm} = \frac{F(+V) - F(-V)}{F(+V) + F(-V)}. \tag{7.2}$$

Boundary-flux terms linear in $K_i E^i$ may be odd under polarity reversal, while scalar energy terms sourced by E^2 are even at leading order. This separates boundary-flux behavior from radion-energy behavior.

Symmetry ratio.

$$R_{\text{sym}} = \frac{F_{\text{asym}}}{F_{\text{matched}}}. \tag{7.3}$$

A true edge-sensitive residual should be suppressed by symmetry-matched dummy devices with comparable mass, voltage, leakage, and thermal behavior.

Inversion ratio.

$$R_{\pi} = \frac{F(\theta + \pi)}{F(\theta)}. \tag{7.4}$$

Rotating the apparatus tests whether the residual is tied to device geometry, support coupling, laboratory gravity, or environmental fields.

Shielding ratio.

$$R_{\text{shield}} = \frac{F_{\text{shielded}}}{F_{\text{unshielded}}}. \tag{7.5}$$

If the residual is ordinary external-field interaction, shielding should suppress it. If it is internal boundary flux, shielding should affect it only through changes in capacitance, grounding, or boundary condition.

8 Coefficient extraction

Let the candidate boundary-flux force be modeled as

$$F_{\text{BF}} = a_{\perp}\mathcal{G}_{\perp} + a_K\mathcal{G}_K + a_{K\Pi}\mathcal{G}_{K\Pi}, \tag{8.1}$$

where the $\mathcal{G}_i$ are geometry functionals computed from field simulations and embedding assumptions. A null bound $|F_{\text{BF}}| < F_{\text{lim}}$ implies

$$|a_{\perp}\mathcal{G}_{\perp} + a_K\mathcal{G}_K + a_{K\Pi}\mathcal{G}_{K\Pi}| < F_{\text{lim}}. \tag{8.2}$$

If devices are designed so that only one functional dominates, the bound simplifies to

$$|a_i| < \frac{F_{\text{lim}}}{|\mathcal{G}_i|}. \tag{8.3}$$

This is the boundary-flux analogue of a fifth-force exclusion curve. It converts a null apparatus into a bound on projection geometry rather than a vague statement that nothing happened.

9 Linearized status and future nonlinear theory

The present derivation is linearized and local. It keeps the leading normal-flux and curvature structures but does not derive a full nonlinear embedded action. A complete version should specify:

1. the five-dimensional action for G_{AB} and $\mathcal{A}_A$;
2. the embedding dynamics of Σ;

3. junction or boundary conditions for Π_μ;
4. the dimensional normalization of a_i;
5. global conservation laws and possible topological terms.

Until these are supplied, the theory should be advertised as an effective boundary-flux framework, not as a finished high-dimensional model.

10 Conclusion

The boundary-flux branch of five-dimensional projection electromagnetism can be made precise enough to test. Pullback protects the homogeneous Maxwell equation. Projection of the source equation naturally introduces normal-flux and embedding-curvature terms. In electrostatics, these terms appear as edge-sensitive effective charge densities and suggest a family of null observables based on pressure, polarity, symmetry, inversion, and shielding. The most important result is methodological: a speculative hypersurface idea becomes publishable when it yields equations that can be bounded even by null experiments.

References

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